ON THE FORM OF THE RELATION BETWEEN RESPONSE RATES IN A MULTIPLE SCHEDULE[1]

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Three pigeons received training on multiple variable-interval schedules with brief alternating components, concurrently with a fixed-interval schedule of food reinforcement on a second key. Fixed-interval performance exhibited typical increases in rate within the interval, and was independent of multiple-schedule responding. Responding on the multiple-schedule key decreased as a function of proximity to reinforcement on the fixed-interval key. The overall relative rate of responding in one component of the multiple schedule roughly matched the overall relative rate of reinforcement. Within the fixed interval, response rate during one multiple-schedule component was a monotonic, negatively accelerated function of response rate during the other component. To a first approximation, the data were described by a power function, where the exponent depended on the relative rate of reinforcement obtained in the two components. The relative rate of responding in one component of the multiple schedule increased as a function of proximity to fixed-interval reinforcement, and often exceeded the overall obtained relative rate of reinforcement. The form of the function relating response rates is discussed in relation to findings on rate-dependent effects of drugs, chaining, and the relation between response rate and reinforcement rate in single-schedule conditions.

The study of operant behavior has been concerned in large measure with steady state relationships between reinforcement schedules and response rates. Such relationships may be characterized as input-output functions, where the output—responding—is some function of the input—reinforcement (cf. Catania and Reynolds, 1968). A good deal of recent work has attempted to identify the simplest and most general specifications of input and output variables, and the form of the function relating them. Although the measures employed and the relations between them have varied from one study to another, some unification of the findings is evident, especially in Herrnstein's (1970) general formulation of the input-output relation, which relates response rate to obtained relative reinforcement.

An alternative characterization of behavioral relations in the steady state is suggested by modern psychophysics. Since the advent of the theory of signal detectability, a great deal of psychophysical research has concentrated on the nature of the detection process, as inferred from the receiver-operating-characteristic or isosensitivity curve (cf. Green and Swets, 1966). The isosensitivity curve is the relation between response probability when one stimulus is presented (say, a white noise), and the response probability when a different stimulus is presented (say, signal plus noise) when responses to the signal are designated as correct. When the stimuli are held constant, the form of the curve has been shown to be invariant with respect to several operations that affect the particular values of the response probabilities. For example, variations in signal probability, relative payoff values, and instructions trace out the same curve (Galanter and Holman, 1967). Thus, its form is presumed to reflect invariances in the process of signal detection.

The central proposal of the present paper is that the study of reinforcement processes may be advanced by a similar analysis. That is, in addition to expanding our knowledge of the independent variables that determine response rates, we should turn our attention to invariances in the relations between response rates.

There are several reasons for pursuing this line of study. The first derives by analogy from signal detection research. The isosensitivity curve of psychophysics can often be character-

ized by an expression with a single free parameter, identified with the subject's sensitivity to the signal. The value of this parameter predicts the behavior of the same observer on related detection tasks (e.g., symmetric forced choice—see Green and Swets, 1966). Consequently, this sensitivity parameter summarizes the behavioral effects of several particular procedures and may itself be studied as a higher-order dependent variable in relation to factors such as signal strength. If the relation between response rates can also be characterized simply, its parameters may likewise be identified with various behavioral processes and have predictive value in other settings, much as the parameters of Herrnstein's formulation are currently identified and used to aid in the understanding of input-output functions.

A second reason also follows from developments in the theory of signal detectability, where the exact form of the isosensitivity curve may facilitate a choice between competing formulations of the detection process (see, for example, Green and Swets, 1966; Luce, 1963). At least two current formulations of maintained operant behavior may be distinguished by the form of the relation between response rates. Lander and Irwin (1968) used the following expression to characterize the relation between response rates and reinforcement rates in a multiple schedule with variable-interval (VI) component schedules:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1{}^a}{r_1{}^a + r_2{}^a} \qquad (1)$$

where R_1 and R_2 represent the rates of responding, and r_1 and r_2 represent the rates of reinforcement, in components 1 and 2 respectively. The exponent a characterizes the sensitivity of the distribution of responding between components to the distribution of reinforcement between components, and the degree of interaction between component schedules. Herrnstein (1970) used a quite different expression to describe the same relation:

$$\frac{R_1}{R_1 + R_2} = \frac{\dfrac{kr_1}{r_1 + mr_2 + r_o}}{\dfrac{kr_1}{r_1 + mr_2 + r_o} + \dfrac{kr_2}{r_2 + mr_1 + r_o}} \qquad (2)$$

where R_1, R_2, r_1, and r_2 are defined as above; the parameter k represents the maximum possible rate of responding under the prevailing

experimental conditions; m represents the degree of interaction between components, much like Lander and Irwin's exponent a; and r_o summarizes the reinforcement obtained by unspecified behaviors other than the designated response. Both Equations 1 and 2 account quite nicely for the existing data (compare, for example, Lander and Irwin's Figure 3 with Herrnstein's Figure 13). However, if both schedules are held constant so that r_1 and r_2 do not change, and the component response rates are made to vary by some means, the predicted relations between response rates may differ. Because the Lander and Irwin formulation contains no terms that determine the level of responding, it must be the case that

$$R_1 = cR_2 \qquad (3)$$

assuming that a is constant. In Herrnstein's formulation, variations in either k or r_o will lead to rate changes in both components. If k changes, while m or r_o remain constant, the relation is the same as expected by the Lander and Irwin account (Equation 3). However, if r_2 is small relative to r_1, R_2 will undergo proportionally greater decreases than R_1 when r_o increases. Thus, if the relation between response rates indicates departure from simple proportionality, with the relative response rate [the left side of Equations (1) and (2)] in the component with the higher rate of reinforcement increasing as the rates go down, Herrnstein's formulation is supported. Equation 2 implies that the relative response rate cannot exceed the relative reinforcement rate [$r_1/(r_1 + r_2)$]—i.e., the matching value—if k, m, and r_o are the same for both schedule components. The present research provides data relevant to these implications of Equations 1 and 2.

Finally, the study of relations between response rates may aid in the isolation of general laws of behavior. The importance of invariant response-response relations in the study of behavior has been emphasized by Galanter (1970). The search for invariances in operant behavior seems especially important at present. Recent demonstrations of the complexity of schedule effects (e.g., Jenkins, 1970), and the partial determination of conventionally studied responses, such as the pigeon's key peck, by factors other than reinforcement contingencies (e.g., Williams and Williams, 1969) have led to some discouragement over the possibility of isolating general laws of operant behavior

(*e.g.*, Herrnstein and Loveland, 1972). The complexity of input-output relations and their specificity to particular species and response systems do not, however, preclude the possibility that the relation between responses studied in two experimental conditions will have a general form.

Consider an experimental situation that is analogous to that employed in signal detectability research, but which permits the study of free-operant behavior in relation to reinforcement schedules. Suppose that two stimuli are presented alternately, each correlated with a particular VI schedule of reinforcement for a free operant (thus constituting a multiple variable-interval variable-interval or *mult* VI VI schedule of reinforcement). The rate of responding in the presence of each stimulus will be determined both by the schedule prevailing in that component, and by the frequency of reinforcement obtained in the alternated component, as described by Herrnstein's (1970) formulation (Equation 2). If, now, both schedules remain unchanged, but some other variable is manipulated that alters the rates of responding in both schedule components, it is possible to construct the analog of an isosensitivity curve by plotting the two independently measured response rates against each other. One such variable is reinforcement for alternative behavior, which will decrease response rates in both components. For example, Pliskoff, Shull, and Gollub (1968) arranged a VI schedule of reinforcement on a second operandum concurrently with a multiple schedule. As the frequency of concurrent reinforcement increased, the rate of responding on the multiple-schedule key decreased in both components, the decrease depending on the value of the VI in the multiple schedule.

The present study may be viewed as a systematic extension of the work of Pliskoff *et al.* The relation between response rates in a *mult* VI VI schedule is studied here by arranging a concurrent fixed-interval (FI) schedule. An FI schedule assures progressive increases in the average rate of responding on the FI key, and inverse rate changes in VI responding within each cycle between FI reinforcements (Nevin, 1971). Thus, different levels of responding may be measured repeatedly within a single session. The method is fundamentally similar to that used by Nevin (1970) to determine isosensitivity curves for detection of a light flash. Uni-

formities in the function relating response rates may be as valuable in characterizing the process of differential reinforcement as are the invariances in isosensitivity curves that describe the process of signal detection.

METHOD

Subjects

Three White Carneaux pigeons having extensive experience with concurrent schedules of reinforcement served as subjects. All three birds had served in a discrete-trial study with concurrent VI schedules (Nevin, 1969), and a free-operant experiment with concurrent FI and VI schedules (Nevin, 1971). For the present study, the birds' free-feeding weights were redetermined and the experiment was conducted with the subjects at 80% of their new weights, ±15 g.

Apparatus

The study was conducted in a two-key Lehigh Valley pigeon chamber, with standard response keys, houselight, and grain feeder. Some masking noise was provided by the chamber blower. Experimental contingencies were arranged by conventional relay circuitry and by Lehigh Valley probability generators. Data were recorded on electromechanical counters. Scheduling equipment was in a separate room from the experimental chamber.

Procedure

Basically, the experiment involved *mult* VI VI schedules of reinforcement for responding on the left key, correlated with red and green keylights, while an FI schedule correlated with a blue keylight was arranged concurrently for responding on the right key. The value of the fixed interval was always 50 sec, timed from the termination of reinforcement for pecking the FI key. Each such FI cycle was divided into 10-sec subintervals, and the left key was lighted red or green in alternate 10-sec subintervals. Because each FI cycle contained an odd number of subintervals, the multiple key would be red in the first subinterval of one cycle, green in the first subinterval of the next cycle, and so forth. In this way, during a daily session of 30 FI cycles, red and green appeared on the multiple key equally often during each successive subinterval of the FI. The key color during the initial cycle of each day's experimentation was selected at random.

Reinforcement was scheduled for pecks on the multiple key in the presence of red and green by a single VI 30-sec tape that operated continuously, with intervals chosen on the basis of the Fleshler-Hoffman (1962) progression. If the currently scheduled interval elapsed while the multiple key was red, reinforcement was assigned with probability p_1 for the next peck on the red key. If the currently scheduled interval elapsed while the multiple key was green, reinforcement was assigned with probability p_2 for the next peck on the green key. Reinforcements remained available until collected by a peck when the key was the appropriate color, even though the VI tape continued to operate, and the other component might intervene between assignment and presentation of the reinforcer. The average interval between reinforcement availabilities per second of red-key time was $30/p_1$, while the average interval between reinforcement availabilities per second of green-key time was $30/p_2$. In effect, the schedules arranged during the multiple-schedule components were random-interval schedules, as arranged by Millenson (1963), except that the intervals between pulses on which reinforcement assignments were based were variable rather than fixed.

The following additional contingencies obtained: (a) At the end of the fifth subinterval in each FI cycle, when reinforcement became available for a peck on the FI key, the color on the multiple key did not change (although 10 sec had elapsed) in order to prevent a correlation between reinforcement for responding on the FI key and color change on the multiple key. Pecks on the multiple key after reinforcement became available on the FI key were recorded separately. (b) A changeover delay (COD) was in effect for switches from one key to the other, such that the first peck on a key following pecking on the other key started a 2.0-sec time interval during which reinforcement was not available, in order to prevent immediate reinforcement for switching. (c) Reinforcement was not available during the first 2.0 sec of each multiple-schedule component, in order to minimize reinforcement of a run of responses initiated during the previous component. (d) Reinforcement consisted of 3.5 sec of access to mixed grain. Only the key that had been pecked to produce reinforcement was darkened during magazine presentation; however, pecks on either key during this time had

no effects. The subinterval timer did not operate during reinforcement.

The above contingencies were developed while the values of p_1 and p_2 were manipulated somewhat unsystematically during the first two months of experimentation with this procedure. By the end of this period, all three birds were performing similarly, with responding on the FI key increasing from near-zero to moderate levels during the course of successive subintervals between FI reinforcements, and differentiated responding on the multiple key in the presence of red and green when the values of p_1 and p_2 differed substantially. At this point, systematic experimentation began. Values of p_1 and p_2 were chosen such that their ratio would be either 5:1 or 2:1, over a range that ensured substantial variation in the obtained rates of reinforcement. Each pair of values remained in effect for 25 consecutive daily sessions, the last five of which provided data for the analyses described below. The values of p_1 and p_2 arranged during each multiple-schedule component, and the expected average intervals between reinforcement availabilities are presented in Table 1. Throughout, sessions began with the first subinterval of the FI cycle, and ended after 30 FI reinforcements were obtained.

Table 1

Values of p_1 and p_2, and expected mean intervals between reinforcement availabilities during successive experimental conditions.

Order of Exposure	(Red) p_1	(Green) p_2	Expected Mean Intervals Between Reinforcements (sec)	
			(Red)	(Green)
1	0.50	0.10	60	300
2	0.40	0.20	75	150
3	0.80	0.40	37.5	75
4	1.00	0.20	30	150
5	0.25	0.05	120	600
6	0.20	0.10	150	300

RESULTS

To illustrate performance on this complex set of reinforcement schedules, one subject and one condition were drawn at random for detailed presentation. For the final five sessions, responses were summed separately for each successive subinterval of the FI according to

which response key was pecked and which color was present on the multiple key. Response rates in the presence of each color were then computed for each subinterval by dividing the total number of key pecks by the total time spent in the presence of that key color at the ordinal subinterval value (750 sec, in all cases). The results of these computations are plotted in Figure 1. Complete data for all subjects and conditions are given in the Appendix.

Certain features of the data presented for this subject and condition are representative of those for all other subjects and conditions. First, the rate of responding on the FI key increased systematically with successive subintervals, and was not affected by the key color (or the rate of responding) on the multiple key. To illustrate the constancy of FI-key performance across multiple-schedule components, Figure 2 presents the rate of responding on the FI key when the multiple key was red in relation to the rate when the multiple key was green, for each ordinal subinterval of the FI. There is no evidence of systematic departure from equality of the response rates, although there is some nonsystematic variation early in the FI, when rates were typically low. Thus, whatever the relation between response rates

on the multiple key, it is not confounded by differential responding on the FI key. This sort of rate constancy in multiple and concurrent schedules has been reported before (Catania, 1962).

Second, the rate of responding on the multiple-schedule key was always higher when the key was red than when the key was green, and both rates decreased systematically with successive subintervals of the FI. When these rates are averaged across all five subintervals, there is an orderly increasing relation between the overall rate of responding when the key was red, relative to the sum of the overall rates of responding to red and green, and the overall relative rate of reinforcement obtained in the presence of red. This relation is presented in Figure 3 for all subjects and conditions. Note that there is considerable variation in the obtained relative frequency of reinforcement, although only two values—0.67 and 0.83—were scheduled. This variation is attributable to two factors: (1) failure of the probability generators to schedule the expected numbers of reinforcements, which is especially likely at low values of reinforcement probability; and (2) failure of the subjects to obtain all scheduled reinforcements, which is especially likely at low response rates.

Most of the data points in Figure 3 fall slightly below the matching line, but the de-
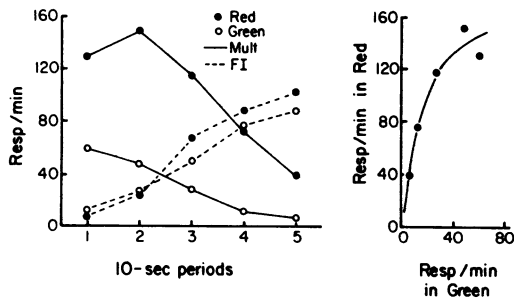

Fig. 1. Performance of Bird #58 during the final five sessions of condition 5, with $p_1 = 0.25$ and $p_2 = 0.05$. In the left-hand panel, the dashed lines show the rate of responding on an FI 50-sec schedule during successive 10-sec periods. Data are plotted separately according to the component of a multiple schedule on a second key. When the second key was red, reinforcement was scheduled every 120 sec on the average; when it was green, reinforcement was scheduled every 600 sec on the average. Red and green alternated every 10 sec. The solid lines show the rate of responding on the multiple key in successive 10-sec periods within the interval between reinforcements on the FI key. The right-hand panel presents the relation between response rates on the multiple key in the presence of red and green throughout the FI.
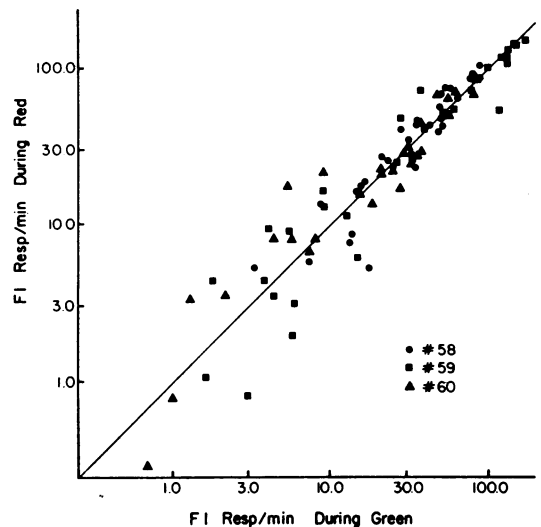

Fig. 2. Response rate on the FI key when the multiple key was red, in relation to response rate on the FI key when the multiple key was green during successive 10-sec subintervals of the FI, for all subjects and conditions. The solid line represents equal responding.
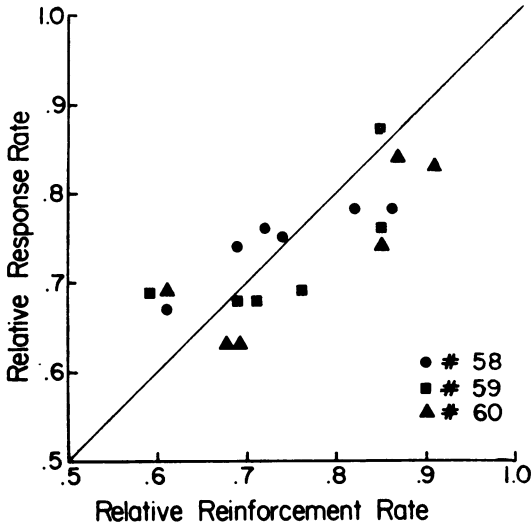
Fig. 3. The rate of responding on the multiple key when it was red, averaged throughout the concurrent FI, relative to the sum of the average rates when the key was red and when it was green, as a function of the obtained relative rate of reinforcement when the key was red. The diagonal line represents matching. Data points are plotted for individual subjects and conditions.

parture from matching is neither large nor consistent across subjects or conditions. Thus, the data are consistent with the findings of Shimp and Wheatley (1971) and Todorov (1972) that relative response rates approximate relative reinforcement rates in multiple schedules with short components. Evidently, the availability of concurrent FI reinforcement and the presence of systematic changes in the absolute rates of multiple-schedule responding within the FI do not disturb this overall matching relation.

Third, Figure 1 suggests that the rate of responding in the presence of red was a monotonic increasing function of the rate of responding in the presence of green, with some evidence of negative acceleration. The relation between response rates is presented for all subjects and conditions in Figure 4, on logarithmic coordinates. The rough linearity of the data suggests that a power function of the form

$$R_1 = qR_2^s \qquad (4)$$

may provide a reasonable first approximation
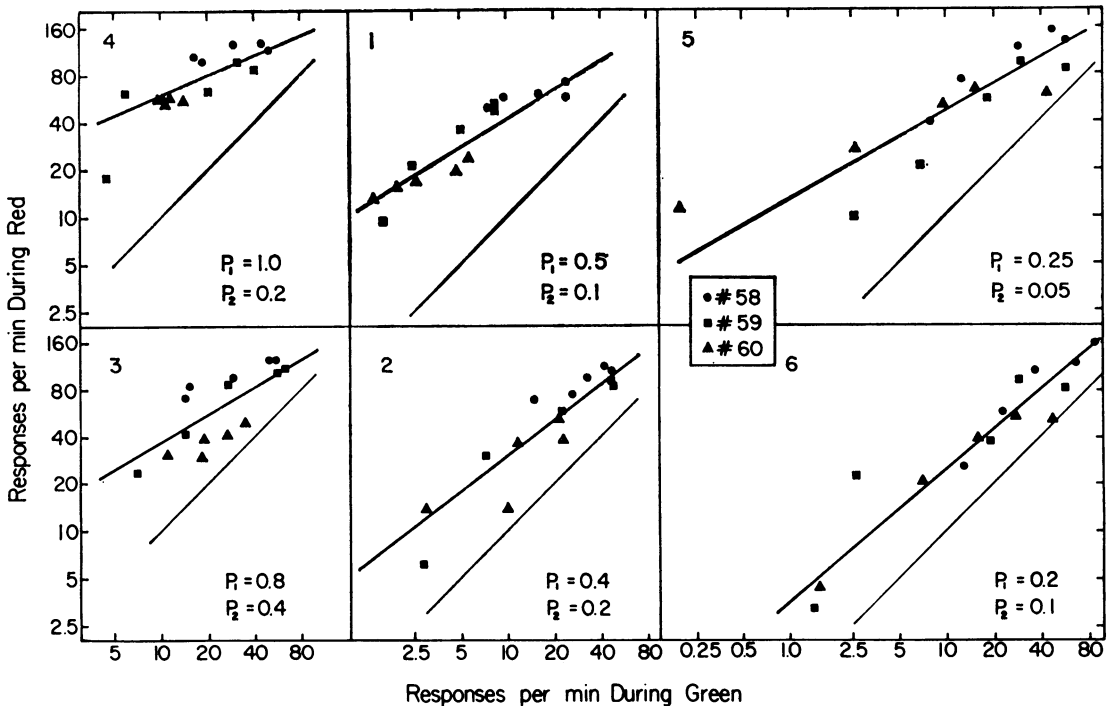
Fig. 4. Response rate on the multiple key when it was red, in relation to response rate on the multiple key when it was green, during successive 10-sec subintervals of the concurrent FI, for all subjects and conditions. The upper panels present data for all schedule conditions where p_1 was five times as great as p_2; for the lower panels, p_1 was twice as great as p_2. Total scheduled rate of reinforcement decreases from left to right. The lower solid line in each panel represents equal responding. The upper line was fitted to the average data as described in the text.

to the relation between response rates. In this equation, R_1 and R_2 represent the rates of key pecking in the presence of red and green, respectively. q is a multiplicative constant representing the rate of responding to red when the rate of responding to green is 1.0 responses per minute; its value is given by the antilog of the intercept of the function in logarithmic coordinates. s is the exponent of the power function, which is given by the slope of the function in logarithmic coordinates.

In each panel of Figure 4, a straight line has been fitted to the data by the following procedure. First, individual multiple-schedule response rates were transformed to common logarithms, and averaged across subjects for each successive subinterval of the FI, separately for red and green. Then, the slope and intercept of the best-fitting linear relation between log response rates were determined for the average transformed data. Because both response rates are subject to error, the usual regression model, which allows for error in only one term, is not appropriate. Accordingly, the slope and intercept were estimated by the procedure described by Isaacs (1970) for the so-called "structural relation" between two variables, both of which are subject to error. Inspection of the data indicated that variability within and across subjects was about equal for responding to red and to green. Therefore, Isaacs' procedure C, with $\lambda = 1$, was employed.

Inspection of Figure 4 suggests that both the slope and the intercept depended on the scheduled reinforcement probabilities. The slopes are always steeper, and the intercepts lower, in the lower panels where the ratio of reinforcement probabilities was 2:1, than in the corresponding panels above them, where the ratio of reinforcement probabilities was 5:1. Figure 5 presents the relations between s, in the upper panel, and q, in the lower panel, and the average obtained relative rate of reinforcement in each condition. It should be clear that if the relative rate of reinforcement affects the overall relative rate of responding, as shown in Figure 3, either q or s or both must change. If the relative rate of reinforcement were 0.5, and the subjects responded identically in the presence of red and green, the values of both parameters would be 1.0. Figure 5 shows that both q and s depart increasingly from 1.0, with q increasing and s decreasing, as the relative rate of reinforcement increases. The inversion in both

functions results from the discrepant data of Bird 59 in Condition 1; otherwise, these functions, which are based on fits to averaged data, are descriptive of the findings for individual subjects.

The finding that the exponent of the power function departs systematically from 1.0 is evidence against any formulation of behavior that implies a simple proportionality of response rates (Equation 3). The finding that the exponent is less than 1.0 when response rate to red is plotted against response rate to green is consistent with Herrnstein's (1970) formulation (Equation 2), with r_0 varied, because it implies that the relative rate of responding to
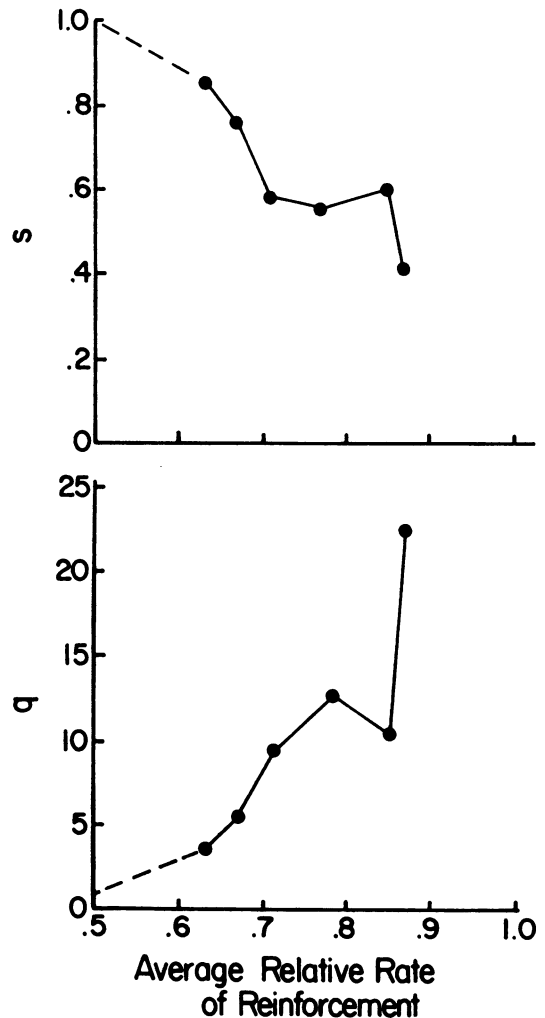

Fig. 5. The relation between the parameters of the power function fitted to the rates of responding on the multiple key (see Figure 4) and the average obtained relative rate of reinforcement in each condition.

red will increase as the absolute rates decrease. However, Herrnstein's formulation sets an upper limit on the rate of responding in one component of a multiple schedule, relative to the sum of rates in the two components. That limit is matching of relative rates of responding and reinforcement. The power relation imposes no such limits: If the exponent is less than 1.0, as shown in Figures 4 and 5, the relative rate of responding increasingly exceeds matching as the absolute response rate decreases.

In this experiment, the absolute rate of responding on the multiple key decreased systematically as a function of successive subintervals between FI reinforcements, and the relative rate of responding during the red component increased. The difference between the relative rate of responding in each subinterval, and the overall relative rate of reinforcement, was calculated for each subject and averaged across subjects for each condition. The average discrepancy between these measures is shown in relation to successive subintervals in Figure 6. Only for $p_1 = 1.0$, $p_2 = 0.2$ do the differences fail to exceed matching, perhaps because this condition sustained the highest overall absolute response rates on the multiple key. In all other conditions, the relative rate of responding in the fifth subinterval exceeded the relative rate of reinforcement, with the sole exception of Bird 59, $p_1 = 0.4$, $p_2 = 0.2$. This tendency to overmatching at low response rates is consistent with the suggestion that responding in one component is a power function of responding in the other.

DISCUSSION

This study sought to determine an analog to the isosensitivity curves of psychophysics by arranging a complex multiple and concurrent schedule of reinforcement that would permit the measurement of different rates of responding repeatedly within single sessions. The present procedure succeeded, in that reasonably orderly relations between response rates were obtained over a wide range of schedule values. The discussion will focus on the extent to which these relations accord with other findings in the literature.

When rate of responding on the multiple key during one component was related to rate of responding on that key during the other
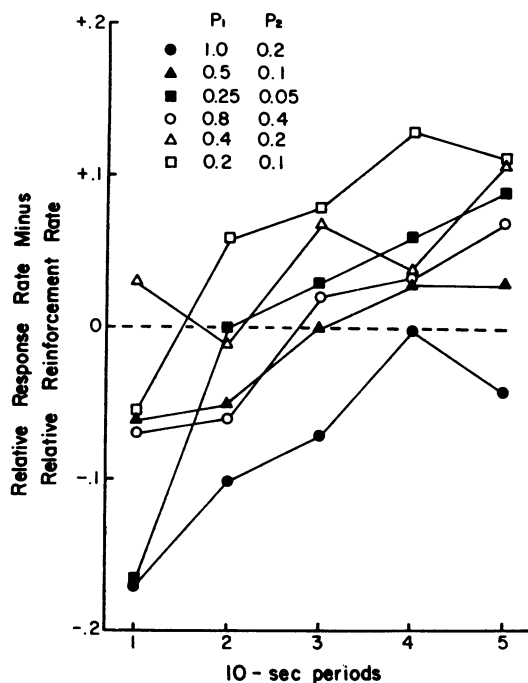

Fig. 6. The average difference between relative response rates and the overall relative reinforcement rates on the multiple key, during successive 10-sec periods of the concurrent FI. The horizontal dashed line indicates matching.

component, the general form of the relation was curvilinear. A power function with an exponent less than 1.0 provided an acceptable first-order description of the relation between response rates, $R_1 = qR_2^s$, where q and s depended on the relative rate of reinforcement.

The power relation implies that as the absolute levels of responding decrease, the relative rate of responding in one component will increase and exceed the relative rate of reinforcement. This implication was confirmed for most of the schedules studied. Although there are no other data known to the author that indicate overmatching of this sort in multiple schedules, overmatching has been observed in post-COD responding in concurrent schedules (Silberberg and Fantino, 1970) and in discrete-trial choice experiments (Shimp, 1966, 1973). A related finding has been reported by Pliskoff *et al.* (1968) in a study employing much the same design and rationale as the present research. In their second experiment, Pliskoff *et al.* studied performance on multiple schedules with component VI schedules arranging 40 and 10 reinforcements per hour, respectively, concurrently with a variable-interval schedule

on a second key. The value of the concurrent VI schedule was varied to give 0, 20, or 60 reinforcements per hour. As the frequency of concurrent reinforcement was increased, the ratio of response rates in the multiple schedule components went up more rapidly than expected on the basis of a model quite analogous to Herrnstein's (1970). It was not, however, possible to determine whether the ratio of response rates would exceed the predicted asymptote of 4.0 (matching).

The apparent overmatching observed in this study may be an effect of changes in obtained rates of reinforcement on the multiple key within the cycle between FI reinforcements. If the obtained relative rate of reinforcement changed during the cycle, the relative rate of responding might exceed the average relative rate of reinforcement, but not its momentary value. The obtained rates of reinforcement were not recorded separately for each subinterval, so this possibility must await further study.

Ideally, the rates of reinforcement should be controlled, rather than measured after the fact. However, the control of obtained relative reinforcement rates in situations involving very low response rates is likely to require considerable modification of the basic schedules, with consequent difficulties for interpretation of the results. Thus, support for the suggested power relation between response rates is more likely to be derived from its generality to several experimental settings than from precise control within a particular procedure.

The power relation between response rates in the two components of the multiple schedule is consistent with the rate-dependent effects of drugs on fixed-interval performance. Kelleher and Morse (1968) and McKearney (1970) have described the effects of drugs on responding in different segments of the FI, and have shown that the increase produced by a drug, relative to the control rate of responding in that segment, is related to the control rate of responding according to a power function of the form:

$$\frac{R_d}{R_c} = kR_c{}^a \qquad (5)$$

where R_c is the control rate of responding, R_d is the rate after drug administration, *a* is a negative exponent, and k is a constant that may depend on the prevailing schedule and stimu-

lus conditions (McKearney, 1970). Equation 5 may be rewritten:

$$R_d = kR_c{}^{(a+1)} \qquad (6)$$

That is, response rate after drug administration is a power function of the control rate. This relation is analogous to that shown in Figure 4 between response rates controlled by the components of a multiple schedule.

A second source of support for the suggested power relation between response rates derives from Fischer and Fantino's (1968) study of chained schedule performance during progressive satiation.[2] These investigators found that the rate of responding in both the initial and terminal links of a chain VI VI schedule decreased geometrically over blocks of reinforcements:

$$R_i = R_{i_o} \cdot b^n$$
$$R_t = R_{t_o} \cdot c^n \qquad (7)$$

where R_i and R_t represent the initial- and terminal-link response rates, R_{i_o} and R_{t_o} represent their initial values, b and c are decay parameters less than 1.0, and n is the ordinal number of blocks of reinforcements. Because both response rates decrease by constant ratios from one block of reinforcements to the next, a power function of the form

$$R_t = kR_i{}^a \qquad (8)$$

is implied, where the constants k and a depend on the initial values of the response rates and the decay parameters.

If a power relation holds generally between response rates determined in successive experimental conditions, it implies a particular form for the relation between the rate of responding and the rate of reinforcement. Consider, for example, an experimental determination of this fundamental input-output function by the following procedure. A given rate of reinforcement is established (r_1) and the rate of responding determined (R_1). The rate of reinforcement is then multiplied by a constant factor to establish a new rate, r_2, and the response rate R_2 is determined. Once again, the rate of reinforcement is multiplied by the constant to give r_3, and the response rate R_3 is determined. If it is the case that $R_2 = q R_1{}^s$, and

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[2] I am indebted to Dr. Peter Killeen for this observation.

that $R_3 = q R_2{}^s$, or, more generally, $R_{n+1} = q R_n{}^s$, the form of the input-output function is readily derived.[3]

First, let R_1, the rate of responding in the first experimental condition, equal c. Then:

$$R_2 = qc^s$$
$$R_3 = q(qc^s)^s = q^{(s+1)}c^{s^2}$$
$$R_4 = q(q^{(s+1)}c^{s^2})^s = q^{(s^2+s+1)}c^{s^3}$$

and, in general:

$$R_{n+1} = q^{\sum_{k=0}^{n-1} s^k} \cdot c^{s^n} \qquad (9).$$

The value of the summation of s^k is $1 - s^n / 1 - s$ (Daniels, 1928), and n, the ordinal number of the experimental condition, is proportional to the logarithm of the rate of reinforcement, because $r_n + 1/r_n$ is constant. Thus, Equation 9 may be written out

$$R = q^{\left(\frac{1 - s^{c' \log r}}{1 - s}\right)} \cdot c^{s^{c' \log r}} \qquad (10).$$

Equation 10 is, then, the general expression for the relation between the rate of responding and the rate of reinforcement, assuming that the power relation holds between response rates in successive experimental conditions, and that q and s, the parameters of the power function, do not change. The equation predicts that as the rate of reinforcement approaches infinity, the rate of responding will approach $q^{\left(\frac{1}{1-s}\right)}$ as an asymptote (since s is less than 1.0). In this respect, then Equation 10 is much like Herrnstein's (1970) equation:

$$R = \frac{kr}{r + r_0} \qquad (11)$$

where k is the asymptotic response rate when r approaches infinity.

The major difference between Equations 10 and 11 is not in their form, or in their ability to account for the available data, but in the assumptions underlying their derivation. For Herrnstein (1970), the relation between rate of responding and rate of reinforcement follows from a statement of the effects of relative rein-

forcement rates in concurrent schedules. In the present account, the relation follows from a general statement of the relation between response rates. It is interesting that these different approaches converge on similar function forms.

REFERENCES

Catania, A. C. Independence of concurrent responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior,* 1962, 5, 175-184.

Catania, A. C. and Reynolds, G. S. A quantitative analysis of the responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior,* 1968, 11, 327-383.

Daniels, F. *Mathematical preparation for physical chemistry.* New York: McGraw-Hill, 1928.

Fischer, K. and Fantino, E. The dissociation of discriminative and conditioned reinforcing functions of stimuli with changes in deprivation. *Journal of the Experimental Analysis of Behavior,* 1968, 11, 703-710.

Fleshler, M. and Hoffman, H. S. A progression for generating variable-interval schedules. *Journal of the Experimental Analysis of Behavior,* 1962, 5, 529-530.

Galanter, E. On the nature of laws in psychology. In J. R. Royce (Ed.), *Toward unification in psychology; the first Banff conference on theoretical psychology.* Toronto, Canada: University of Toronto Press, 1970.

Galanter, E. and Holman, G. L. Some invariances of the isosensitivity function and their implications for the utility function of money. *Journal of Experimental Psychology,* 1967, 73, 333-339.

Green, D. M. and Swets, J. A. *Signal detection theory and psychophysics.* New York: Wiley, 1966.

Herrnstein, R. J. On the law of effect. *Journal of the Experimental Analysis of Behavior,* 1970, 13, 243-266.

Herrnstein, R. J. and Loveland, D. H. Food-avoidance in hungry pigeons, and other perplexities. *Journal of the Experimental Analysis of Behavior,* 1972, 18, 369-383.

Isaacs, P. D. Linear regression, structural relations, and measurement error. *Psychological Bulletin,* 1970, 74, 213-218.

Jenkins, H. M. Sequential organization in schedules of reinforcement. In W. N. Schoenfeld (Ed.), *The theory of reinforcement schedules.* New York: Appleton-Century-Crofts, 1970. Pp. 63-109.

Kelleher, R. T. and Morse, W. H. Determinants of the specificity of behavioral effects of drugs. *Ergebnisse der physiologie,* 1968, 60, 1-56.

Lander, D. G. and Irwin, R. J. Multiple schedules: effects of the distribution of reinforcements between components on the distribution of responses between components. *Journal of the Experimental Analysis of Behavior,* 1968, 11, 517-524.

Luce, R. D. Detection and recognition. In R. D. Luce, R. R. Bush, and E. Galanter (Eds.), *Handbook of mathematical psychology,* Vol. 1, New York: Wiley, 1963. Pp. 103-189.

McKearney, J. W. Rate-dependent effects of drugs: modification by discriminative stimuli of the effects

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of amobarbital on schedule-controlled behavior. *Journal of the Experimental Analysis of Behavior,* 1970, **14,** 167-175.

Millenson, J. R. Random-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior,* 1963, **6,** 437-443.

Nevin, J. A. Interval reinforcement of choice behavior in discrete trials. *Journal of the Experimental Analysis of Behavior,* 1969, **12,** 875-885.

Nevin, J. A. On differential stimulation and differential reinforcement. In W. C. Stebbins (Ed.), *Animal psychophysics.* New York: Appleton-Century-Crofts, 1970. Pp. 401-423.

Nevin, J. A. Rates and patterns of responding with concurrent fixed-interval and variable-interval reinforcement. *Journal of the Experimental Analysis of Behavior,* 1971, **16,** 241-247.

Pliskoff, S. S., Shull, R. L., and Gollub, L. R. The relation between response rates and reinforcement rates in a multiple schedule. *Journal of the Experimental Analysis of Behavior,* 1968, **11,** 271-284.

Shimp, C. P. Probabilistically reinforced choice behavior in pigeons. *Journal of the Experimental Analysis of Behavior,* 1966, **9,** 443-455.

Shimp, C. P. Probabilistic discrimination learning in the pigeon. *Journal of Experimental Psychology,* 1973, **97,** 292-304.

Shimp, C. P. and Wheatley, K. C. Matching to relative reinforcement frequency in multiple schedules with a short component duration. *Journal of the Experimental Analysis of Behavior,* 1971, **15,** 205-210.

Silberberg, A. and Fantino, E. Choice, rate of reinforcement, and the changeover delay. *Journal of the Experimental Analysis of Behavior,* 1970, **13,** 187-197.

Todorov, J. C. Component duration and relative response rates in multiple schedules. *Journal of the Experimental Analysis of Behavior,* 1972, **17,** 45-49.

Williams, D. R. and Williams, H. Auto-maintenance in the pigeon: sustained pecking despite contingent non-reinforcement. *Journal of the Experimental Analysis of Behavior,* 1969, **12,** 511-520.

APPENDIX

Responses per minute in the presence of red and green keylights on the multiple key, and on the concurrently available FI key, during successive 10-sec subintervals between reinforcements on the FI key. Data are pooled for the final five sessions of training on each experimental condition. Also given are total reinforcements obtained in the presence of red and green on the multiple and FI keys during the final five sessions of training.

Condition 1 $p_1 = 0.5$, $p_2 = 0.1$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	*2*	*3*	*4*	*5*		*1*	*2*	*3*	*4*	*5*	
#58	Mult	57.0	72.1	60.9	56.9	49.6	77	24.2	24.5	16.2	9.7	7.9	17
	FI	5.8	16.2	25.3	28.1	23.2	75	7.2	14.8	22.9	31.0	32.6	75
#59	Mult	47.8	52.8	36.2	21.4	9.5	77	8.5	8.5	5.1	2.5	1.6	14
	FI	1.0	9.2	24.8	51.6	71.4	75	1.6	5.6	26.6	53.4	78.0	75
#60	Mult	23.7	19.5	16.8	15.5	13.1	72	5.8	5.4	2.6	2.0	1.4	11
	FI	3.4	8.2	13.6	21.2	25.1	75	1.3	8.0	18.7	24.8	32.9	75

Condition 2 $p_1 = 0.4$, $p_2 = 0.2$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	*2*	*3*	*4*	*5*		*1*	*2*	*3*	*4*	*5*	
#58	Mult	113.8	108.0	96.2	76.6	69.5	45	43.3	46.8	32.6	26.8	15.0	20
	FI	13.9	27.4	46.2	41.7	42.3	75	8.8	21.2	35.1	41.8	51.5	75
#59	Mult	93.2	85.6	58.3	30.2	6.2	46	47.0	49.0	24.9	7.4	2.9	19
	FI	4.5	13.2	40.2	100.9	142.3	75	3.8	9.1	39.0	98.5	140.4	75
#60	Mult	53.8	39.2	36.7	14.3	14.1	46	21.6	23.4	11.7	10.1	3.0	29
	FI	0.7	17.9	29.3	68.6	69.3	75	3.0	5.4	36.8	47.8	81.3	75

Condition 3 $p_1 = 0.8$, $p_2 = 0.4$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	2	3	4	5		1	2	3	4	5	
#58	Mult	121.0	120.7	95.2	85.7	70.6	115	54.6	50.2	29.4	15.3	14.6	41
	FI	8.8	18.9	34.6	39.4	47.7	75	13.8	16.6	30.6	48.1	49.8	75
#59	Mult	102.2	108.0	86.3	41.8	23.2	110	56.6	64.6	27.3	14.4	7.0	50
	FI	1.9	9.5	26.5	89.4	114.2	75	5.7	4.1	32.6	84.3	128.2	75
#60	Mult	49.4	40.3	39.6	29.8	30.2	103	35.6	27.3	19.1	18.2	11.3	47
	FI	6.9	21.8	28.9	45.4	50.5	75	7.3	9.1	33.0	36.8	56.1	75

Condition 4 $p_1 = 1.0$, $p_2 = 0.2$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	2	3	4	5		1	2	3	4	5	
#58	Mult	112.3	124.9	121.0	102.3	97.8	138	45.2	45.4	30.0	17.0	19.0	23
	FI	5.2	41.0	56.0	64.6	75.8	75	17.8	27.8	49.1	62.7	57.3	75
#59	Mult	85.2	98.0	64.3	60.2	17.9	138	41.0	32.1	20.4	5.4	4.5	25
	FI	1.8	6.6	48.4	53.7	116.0	75	4.4	15.1	28.0	113.6	119.8	75
#60	Mult	55.4	56.6	56.6	52.9	55.8	155	14.3	11.7	10.8	11.0	9.9	16
	FI	8.2	15.9	21.2	27.4	30.4	75	5.8	15.8	21.1	29.4	31.4	75

Condition 5 $p_1 = 0.25$, $p_2 = 0.05$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	2	3	4	5		1	2	3	4	5	
#58	Mult	129.6	148.6	114.6	71.8	38.8	26	59.5	48.1	29.3	12.6	8.2	10
	FI	7.8	23.2	67.6	87.5	101.8	75	13.2	25.6	49.4	76.9	88.3	75
#59	Mult	85.4	94.8	54.5	20.3	9.8	29	58.8	30.2	18.5	6.9	2.6	9
	FI	3.1	16.5	54.8	126.6	149.6	75	5.9	8.9	59.8	129.1	168.3	75
#60	Mult	58.5	61.7	50.1	26.4	11.2	28	45.0	15.7	9.9	2.7	0.2	5
	FI	0.7	8.4	17.8	64.5	82.5	75	1.0	4.4	27.7	54.6	83.8	75

Condition 6 $p_1 = 0.2$, $p_2 = 0.1$

		Multiple Key Red R/min					Total Rfts	Multiple Key Green R/min					Total Rfts
		1	2	3	4	5		1	2	3	4	5	
#58	Mult	118.7	153.9	102.6	48.3	24.9	25	65.6	86.2	35.9	22.2	12.3	16
	FI	5.4	18.2	46.1	72.5	93.5	75	3.3	15.8	35.4	53.0	76.5	75
#59	Mult	80.2	92.3	36.6	22.5	3.2	17	56.2	28.4	18.7	2.6	1.4	12
	FI	3.5	11.7	72.4	105.6	149.3	75	4.3	12.7	37.0	131.5	143.1	75
#60	Mult	51.4	53.4	39.4	20.4	4.4	21	46.5	27.2	15.3	6.9	1.5	10
	FI	0.3	3.6	23.6	70.0	82.6	75	0.7	2.3	21.3	62.0	81.3	75